

Integrations: Integration is the reverse process of differentiation.

The literal meaning of the word integration is summation. i.e. it is the process of finding the limit of a sum of a certain number of terms say n .

Some basic questions:

Q1) Evaluate $\int \tan^2 x dx$.

Solution, $\int \tan^2 x dx$

$$= \int (\sec^2 x - 1) dx$$

$$[\because \sec^2 x - \tan^2 x = 1]$$

$$= \tan x - x + C$$

Q2) i) $\int \sqrt{1 + \sin^2 x} dx$.

Solution,

$$y = \sqrt{(1 + \sin^2 x)}$$

$$= \sqrt{\sin^2 x + \cos^2 x + 2\sin^2 x \cos x}$$

$$= \sqrt{(\sin x + \cos x)^2}$$

$$= \sin x + \cos x$$

$$= -\cos x + \sin x$$

$$= \sin x - \cos x + C$$

ii) $\int \sqrt{1 + \cos^2 x} dx$

Solution:

$$\int \sqrt{1 + \cos^2 x} dx$$

$$\because \sqrt{1 + \cos^2 x} = \sqrt{1 + \cos^2 x}$$

$$\begin{aligned} \cos^2 \theta &= 1 + \cos 2\theta \\ \sin^2 \theta &= 1 - \cos 2\theta \end{aligned}$$

1. $\sqrt{2} \cos x$
2. $\sqrt{2} \sin x + C$ Ans

Q. 4)

Evaluate

$$\int \frac{x(x+3)}{(x+5)} dx$$

Solution

$$\int \frac{x(x+3)}{(x+5)}$$

$$= \int \frac{x(x+5-2)}{(x+5)}$$

$$= \int \left[\frac{x(x+5)}{(x+5)} - \frac{2x}{x+5} \right]$$

$$= \int \left[x - \frac{2x}{x+5} \right]$$

$$= \int \left[x - \frac{2(x+5)-5}{x+5} \right]$$

$$= \int \left[x - 2 + \frac{5}{x+5} \right]$$

On integrating we get

$$= x - 2 + \frac{5}{x+5}$$

$$= \frac{x^2}{2} - 2x + 5 \ln|x+5| + C$$

Q) Evaluate $\int \sin^3 x \, dx$

o.p we know that

$$\sin^3 x = 3 \sin x - 4 \sin^3 x$$

$$\therefore \sin^3 x = \frac{3 \sin x}{4} - \frac{\sin^3 x}{4}$$

$$= \frac{-8}{4} \cos x + \frac{\cos x}{12}$$

$$= \frac{1}{12} (\cos 3x - 9 \cos x) \quad \underline{\text{Ans}}$$

Q7) $\int \sin^2 \frac{x}{2} dx$
solution,

Or using $\int \sin^2 x dx$

$$= \int \left(\frac{1 - \cos 2x}{2} \right)^2 dx$$

$$= \int \left(\frac{1 - \cos 2x}{2} \right)^2 dx$$

$$= \frac{1}{4} (1 - 2 \cos 2x + \cos^2 2x)$$

$$= \frac{1}{4} \left(1 - 2 \cos 2x + \frac{1 + \cos 4x}{2} \right) dx$$

$$= \frac{1}{4} \left(x - \frac{2 \sin 2x}{2} + \frac{1}{2} \left(x + \frac{\sin 4x}{4} \right) \right)$$

$$= \frac{1}{4} \left(x - \sin 2x + \frac{1}{2} x + \frac{\sin 4x}{4} \right)$$

$$= \frac{1}{4} \left(\frac{3x}{2} - \sin 2x + \frac{\sin 4x}{4} \right)$$

Ans